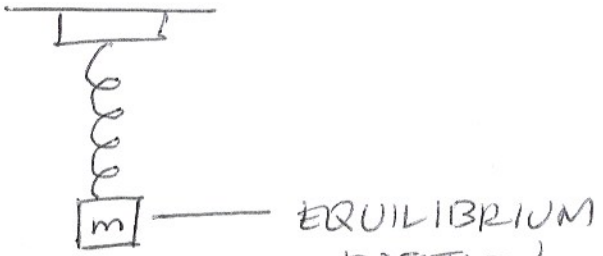
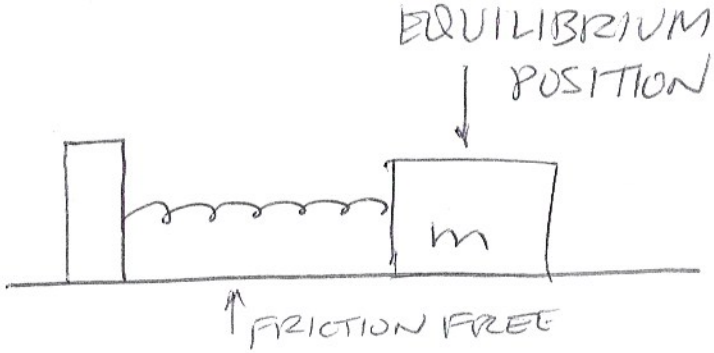
MASS SPRING SYSTEMS

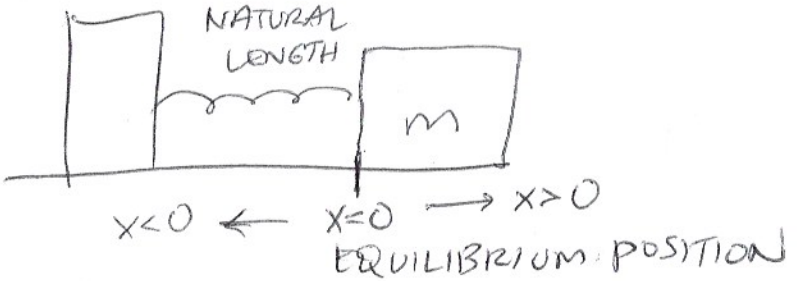




NEWTON'S LAW

$$\vec{F} = m\vec{a}$$

$$\vec{F} = \sum \vec{F}_i$$



$F_{SPR} \propto$ LENGTH, SPRING IS STRETCHED/COMPRESSED

$F_{SPR} = -kx, \; k>0$

IF $x>0$, SPRING STRETCHED
FORCE OF SPRING F TO LEFT
$F<0$

IF $x<0$, SPRING COMPRESSED
FORCE OF SPRING F TO RIGHT
$F>0$

$$ma = -kx$$

$$mx'' = -kx$$

$$mx'' + kx = 0$$

$mx'' + kx = 0 \qquad m, k > 0$

$mr^2 + k = 0$

$r = \pm\sqrt{\frac{k}{m}}\, i \longrightarrow x = C_1 \cos\sqrt{\frac{k}{m}}t + C_2 \sin\sqrt{\frac{k}{m}}t$

$x = A\cos\left(\sqrt{\frac{k}{m}}t + \phi\right)$ FROM MATH 42

$\text{PERIOD} = \frac{2\pi}{\sqrt{\frac{k}{m}}} = 2\pi\sqrt{\frac{m}{k}}$

AS m INCREASES, PERIOD INCREASES CONFIRMED $\frac{m}{k} \uparrow$ $2\pi\sqrt{\frac{m}{k}} \uparrow$

AS k INCREASES, PERIOD DECREASES CONFIRMED $\frac{m}{k} \downarrow$ $2\pi\sqrt{\frac{m}{k}} \downarrow$
(STIFFER SPRING)

UNREALISTIC BECAUSE

MASS OSCILLATES FOREVER

BETWEEN $-A$ AND A

$F_{FLUID\ OR\ AIR} = -bx'$, $b>0$

$$mx'' = -bx' - kx$$

$$mx'' + bx' + kx = 0 \quad m, b, k > 0$$

$$mr^2 + br + k = 0$$

$$r = \frac{-b \pm \sqrt{b^2-4mk}}{2m}$$

① $r = r_1, r_2 \in \mathbb{R}$ AND $r_1 \neq r_2$

② $r = r_1, r_1 \in \mathbb{R}$

③ $r = \alpha \pm \beta i$ $\alpha, \beta \in \mathbb{R}$

① $r = r_1, r_2 \in \mathbb{R}$, $r_1 \neq r_2$ $\quad r_1, r_2 > 0$

$x = c_1 e^{-r_1 t} + c_2 e^{-r_2 t}$ (ASSUME $r_1 > r_2$)

SANITY CHECK: AS $t \to \infty$, $x \to c_1 \cdot 0 + c_2 \cdot 0 = 0$

$b^2 > b^2 - 4mk > 0$

$\sqrt{b^2} > \sqrt{b^2-4mk}$

$b > \sqrt{b^2-4mk} \longrightarrow -b < -\sqrt{b^2-4mk}$

$-b \pm \sqrt{b^2-4mk}$ $\quad -b + \sqrt{b^2-4mk} < 0$

$-b + \sqrt{b^2-4mk} < 0$ $\quad -b - \sqrt{b^2-4mk} < 0$

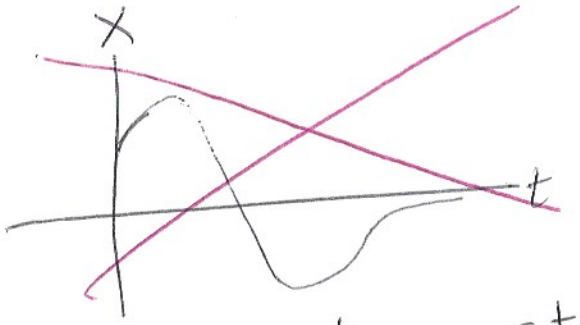
$b^2 - 4mk > 0$

$b^2 > 4mk$

$b > 2\sqrt{mk}$

↑

STRONG FLUID RESISTANCE ⟶ FINITE OSCILLATION ?



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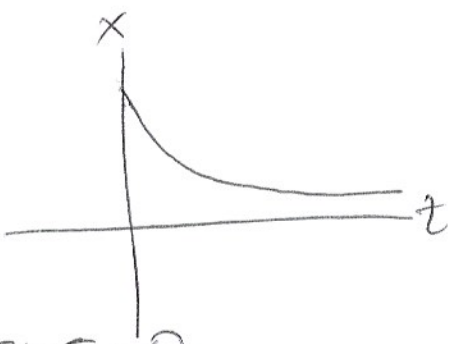

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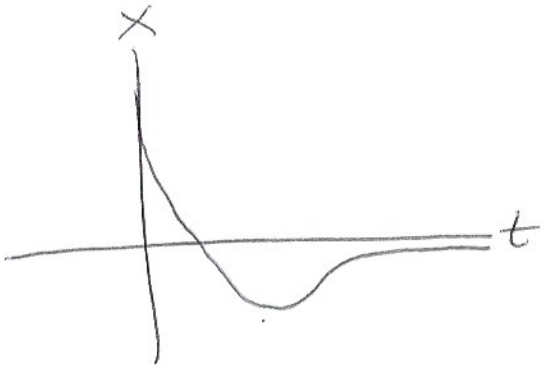
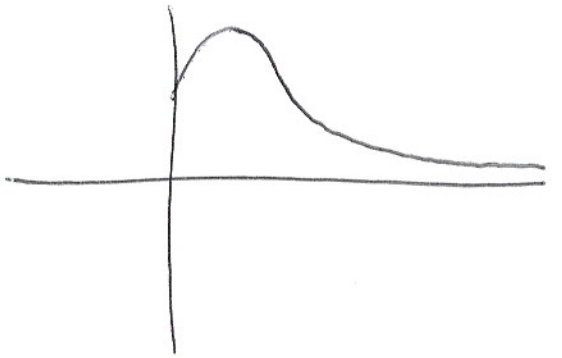


$x = c_1 e^{-r_1 t} + c_2 e^{-r_2 t} \qquad r_1 > r_2 > 0$

$x' = -c_1 r_1 e^{-r_1 t} - c_2 r_2 e^{-r_2 t} = 0$

$e^{r_1 t}(-c_1 r_1 e^{-r_1 t}) = (c_2 r_2 e^{-r_2 t}) e^{r_1 t} \qquad e^{r_1 t} \neq 0$

$-c_1 r_1 = c_2 r_2 e^{(r_1 - r_2)t}$



$$\frac{1}{r_1 - r_2} \ln\left(\frac{-c_1 r_1}{c_2 r_2}\right) = t$$

IF $c_1, c_2 > 0$, $t \notin \mathbb{R}$ $\longrightarrow$ NO LOCAL EXTREMA

IF $c_1, c_2 < 0$, $t \notin \mathbb{R}$ $\longrightarrow$ "

IF $c_1 > 0, c_2 < 0$
OR $c_1 < 0, c_2 > 0$ $\qquad -\frac{c_1 r_1}{c_2 r_2} > 0$

IF OR $\frac{-c_1 r_1}{c_2 r_2} < 1$, $t < 0$ $\longrightarrow$ NO LOCAL EXTREMA

IF $\frac{-c_1 r_1}{c_2 r_2} > 1$ $\longrightarrow$ t HAS 1 REAL POSITIVE VALUE

1 LOCAL EXTREMA